Nonequilibrium critical dynamics of the triangular antiferromagnetic Ising model

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We investigate the nonequilibrium critical dynamics of antiferromagnetic Ising model on a two-dimensional triangular lattice via dynamic Monte Carlo simulation employing spin-flip kinetics. Macroscopic degeneracy of the ground state originating from geometric frustration fundamentally affects the nonequilibrium dynamics of the system. In particular, the defects and the loose spins (whose flip costs no energy) play key roles in the dynamics. The long-time evolution is characterized by a critical dynamic scaling with a growing length scale $\xi(t)$. With random initial configurations, $\xi(t)$ exhibits a subdiffusive growth in time, $\xi(t)\sim t^{1/z}$ with $1/z \simeq 0.43$, while $\xi(t)$ shows a diffusive growth with $z=2$ for the relaxation within the dominant sector of the ground-state manifold. The nonequilibrium critical dynamics therefore exhibits an interesting initial-state dependence. Persistence and the two-time temporal properties are also discussed.

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I. INTRODUCTION

Antiferromagnetic Ising (AFI) model on a triangular lattice, a prime example of periodically frustrated Ising models [1], has remarkable equilibrium properties originating from frustration caused by the geometry of the underlying lattice. The model has an exponentially large number of ground-state degeneracies, leading to nonvanishing entropy per spin at zero temperature [2,3]. Moreover, the ground-state is critical in that the equilibrium spin-correlation function $C(r)$ shows an algebraic decay with their separation r, i.e., $C(r) \sim r^{-\eta}$ with $\eta=1/2$ [4–6]. With higher spins, further interesting features have been observed [7,8]. Recently there has been a renewed interest on the triangular AFI model (with elastic distortions [9–12] or a staggered field [13] to lift the infinite degeneracy) in the context of nonequilibrium glassy dynamics which may offer insights into the nature of the liquid-glass transition.

One may expect that this macroscopic ground-state degeneracy would crucially affect the nonequilibrium evolution process followed by a quench to the zero (critical) temperature. In this work, we aim to characterize this nonequilibrium critical dynamics of the triangular AFI model using a dynamic Monte Carlo simulation method with spin-flip kinetics. We find that the system exhibits a unique coarsening dynamics due to an exponentially large number of degeneracies and criticality of the ground-state. We first consider the dynamics evolved from a random disordered initial state in which there exist many localized (point) defects. In this situation, the coarsening process is governed by defect dynamics on the fluctuating background of ground-state configurations. These ground-state fluctuations are mediated by the loose spins whose flips require no energy cost. The coarsening manifests a single growing length scale. In particular, the average defect separation and the spin-correlation length show the same subdiffusive growth in time $R(t)\sim t^{\phi}$ with $\phi\sim 0.43$. Moreover, the root-mean-square (RMS) displacement of an isolated defect also gives the same subdiffusive growth. In addition, the coarsening exhibits a dynamic self-similarity under the spatial rescaling, which is demonstrated by the critical dynamic scaling of the appropriate spin-correlation function.

As discussed in detail below, our simulation results (in particular, the subdiffusive growth law) appear to be in disagreement with the earlier simulation study of Moore *et al.* [14] on the defect dynamics in the present system. In their study, Moore *et al.* contend that the defects are interacting via entropic Coulomb force, and as a result the defect density decays in time as $\rho(t)\sim(\ln t)/t$. This implies that the growing length scale exhibits a diffusive growth with logarithmic correction, just as in the coarsening dynamics of the two-dimensional XY model.

A close examination on the defect motion seems to offer a different viewpoint on the defect dynamics. We observe that the defect motion is highly correlated with fluctuations of the loose spins; that is, depending on the nearby spin configurations, defects are not allowed to move in arbitrary directions but are constrained to take only limited number of directions. Moreover, the fact that the RMS displacement of an isolated defect exhibits the same subdiffusive behavior as the growing length scale suggests that defects perform a kind of restricted random walk in the *absence* of mutual interaction force. We believe that it is this correlated motion of defects with loose spins in the fluctuating background configurations that gives rise to the observed subdiffusive growth. We also find that the defect motion becomes more subdiffusive when fluctuations of loose spins are forced to be frozen-in.

Starting from a defect-free initial state and quenching the system to the zero temperature, one can look at the dynamic evolution within the ground-state manifold. It is well known [15] that a useful way of classifying the degenerate ground-states is to map each ground-state to its corresponding dimer configuration. Strings of dimers can be obtained from an overlap of one dimer configuration with the given reference (see below for details). The number of ground-states belonging to a given string density has been exactly computed [13] and hence the ground-state manifold is divided into sectors each of which has a unique string density. The dynamics in this case proceeds only via flips of the loose spins. Therefore, the initial state without the loose spin cannot relax and the

dynamics becomes trivially nonergodic. Also the string density is shown to be conserved during the time evolution. Hence the evolving states should be confined within the very sector to which the initial state belongs. The dynamics within the ground-state manifold is therefore nonergodic. In the present work, we have focused on the dynamics within one particular sector which has the string density $p=2/3$. This sector is dominant over other sectors in its number of ground-states: it is the maximum entropy sector [13]. In fact, starting from a random disordered state, the system evolves into this dominant sector. The relaxation within this sector also shows a critical dynamic scaling property. But, in contrast to the dynamics from random initial state with defects, the correlation length of spin fluctuations grows *diffusively* in time: $\xi(t)\sim t^{1/2}$. The nonequilibrium critical dynamics in the present system therefore exhibits an initial-state dependence. A similar initial-state dependence has been recently observed in the nonequilibrium critical dynamics of the two-dimensional XY model [16]. However, one distinct feature in the present system is that the growing length scale exhibits power-law growth for both disordered (with defects) and ordered (without defects) initial states with two different growth exponents. In contrast to the case of the two-dimensional XY model, there is no logarithmic correction in the present system.

We also discuss nonequilibrium properties of aging and persistence. Particularly interesting is the aging dynamics exhibited by the two-time spin-autocorrelation function. In usual kinetic Ising models, the relaxation time τ_r is observed to grow linearly with the waiting time t_w; $\tau_r\sim t_w$. But in the present system, the relaxation becomes slower, leading to a sublinear dependence on waiting time; $\tau_r\sim t_w^{\mu}$ with $\mu\simeq 0.89$.

After introducing the model and simulation method, we discuss details of the above statements. The paper ends with summary and concluding remarks.

II. THE MODEL AND SIMULATION METHOD

The Hamiltonian of the AFI model on a triangular lattice in two dimensions is given by

$$H=J\sum_{\langle ij\rangle}\sigma_i\sigma_j, \tag{1}$$

where $\sigma_i=\pm 1$ is the Ising spin at site i, $J>0$ the interaction strength (we set $J=1$ throughout the paper), and $\langle ij\rangle$ denotes the nearest-neighbor pairs with the coordination number being equal to six in a triangular lattice. We perform dynamic Monte Carlo simulations on the model subjected to $T=0$ quench using a single spin-flip Metropolis dynamics with the flip-probability

$$A=\begin{cases}0 & \text{for } \delta E>0\\ 1 & \text{for } \delta E\leqslant 0,\end{cases} \tag{2}$$

where δE is the energy difference between before and after the flip of a randomly chosen spin. Note that since we are considering zero-temperature quench only, the flip with positive energy cost is always rejected. In this work, we are considering the two different initial configurations. The first one is the random initial configurations in which many defects are generated. For this initial state, the time evolution is characterized by the annihilations of defect pairs. The other initial state is one of the ground-states which belong to the dominant ground-state sector. We use the lattice with linear dimension $L=3000$ for the case of random initial state, and $L=1020$ for that of ordered initial state employing periodic boundary conditions (the total number of spins is given by $N=L\times L$).

III. SIMULATION RESULTS AND DISCUSSION

The system can be divided into three sublattices. It is well known [4] that the equilibrium spatial correlation between spins separated by the distance r which belongs to the same sublattice exhibits an algebraic decay with r, i.e., $C_{eq}(r)\sim r^{-\eta}$ with $\eta=1/2$. We thus first consider the nonequilibrium spatial spin correlation in the *same* sublattice [17]:

$$C(r,t)=\frac{1}{N/3}\left\langle\sum_{i=1}^{N/3}\sigma_i(t)\sigma_{i+r}(t)\right\rangle, \tag{3}$$

where $\sigma_i(t)$ is the spin at site i at time t which is measured from the time of the quench and $\langle\cdots\rangle$ denotes an average over different initial states. In the simulation, the r dependence of $C(r,t)$ for a given time t is measured along the three major axes of the triangular lattice with $r=3na_0$ where $n=0,1,2,\ldots$ and a_0 is the lattice spacing. The final result of $C(r,t)$ for a given sublattice is the average of these three values.

Our simulation shows essentially the same results for three different sublattices and the presented result in Fig. 1(a) is the averaged result over three sublattices for better statistics. This dynamic spin-correlation function reveals the existence of a growing length scale with time. Due to the criticality of the ground-state one may expect that the dynamic spin-correlation function exhibits a critical dynamic scaling of the form

$$C(r,t)=r^{-\eta}F(r/\xi(t)),\quad \eta=1/2. \tag{4}$$

In order to test this scaling, we first extract a length scale $\xi(t)$ from $\tilde{C}(r,t)\equiv r^{\eta}C(r,t)$ with $\eta=1/2$ for a given time t, which is defined as $\tilde{C}(r=\xi(t),t)=C_0$, C_0 being an arbitrary constant. Here we take $C_0=0.2$. Shown in Fig. 1(a) is the plot $\tilde{C}(r,t)$ versus the rescaled distance $r/\xi(t)$ for different times $t=10\times 2^n$ Monte Carlo steps(MCS), with $n=2,3,\ldots,13$. All curves are collapsed onto a single master curve. Worse collapse in the rescaled distance $r/\xi(t)$ larger than ~ 2 seems to be due to the fact that the multiplying factor r^{η} in the scaling scheme has an effect of magnifying the small correlation region in the long distances. As shown in Fig. 1(b), the nonequilibrium spin-correlation length $\xi(t)$ exhibits a power-law growth in time as $\xi(t)\sim t^{1/z}$. The slope in Fig. 1(b) gives $1/z\simeq 0.43(1)$ $[z\simeq 2.33(5)]$.

In order to understand better this subdiffusive coarsening behavior, we focus on the structure of the ground-state and

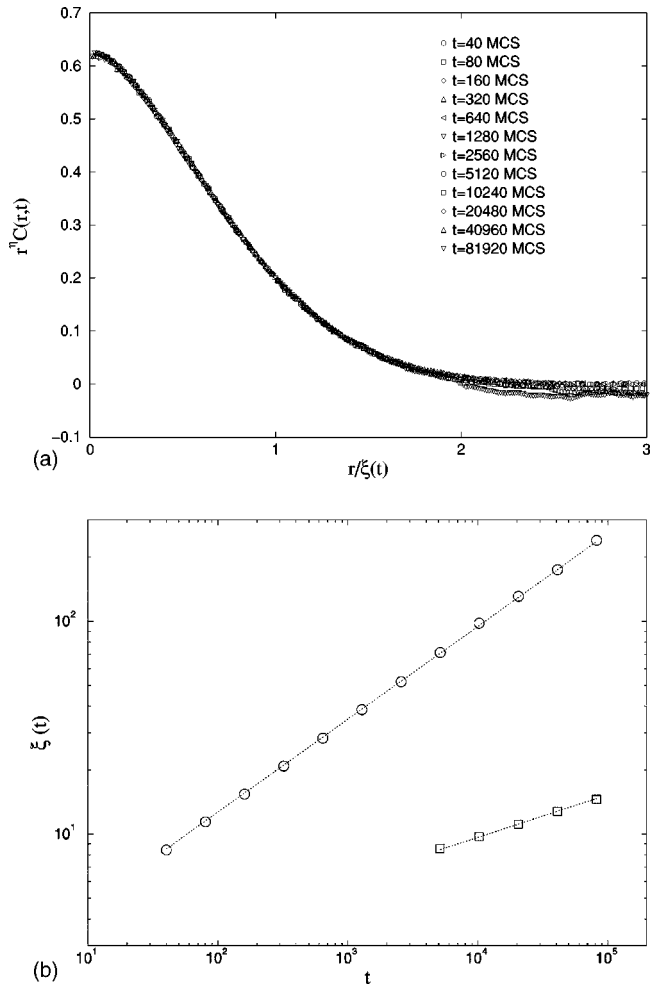

FIG. 1. (a) A scaling collapse for $C(r,t)$ using a critical dynamic scaling $C(r,t)=r^{-\eta}F(r/\xi(t))$ with $\eta=1/2$. This scaling property demonstrates that the evolution process possesses a dynamic self-similarity when spatial distance is rescaled with respect to the nonequilibrium spin-correlation length $\xi(t)$ which is defined as $\tilde{C}(\xi(t),t)=0.2$ with $\tilde{C}(r,t)\equiv r^{\eta}C(r,t)$. (b) Growth of the spin-correlation length $\xi(t)$ in a double-log plot of $\xi(t)$ vs t. The line on the left (circles) is for the case in which the background configuration is fluctuating, and the one on the right (squares) is for the case in which the background state is frozen. Dotted lines are straight lines with slopes 0.43 and 0.18. Note that in the case of frozen background, the length $\xi(t)$ is measured for $t=5120$, 10 240, 20 480, 40 960, 81 920 MCS due to slower growth.

lowest excitation (defect) in the system. Due to the geometry of the triangular lattice, the ground-state is characterized by a spin configuration in which each unit triangular plaquette inevitably has one and only one ''unsatisfied'' (ferromagnetic) bond. It is then easy to see qualitatively why there must be exponentially large number of degeneracies in the ground-state [2]. Let us first denote σ_α as a spin belonging to the sublattice $\alpha=A,B$, and C. Note then that six nearest neighbors of any spin σ_A cannot belong to the sublattice A. We first make the signs of all σ_A's and σ_B's opposite. This assignment automatically leads to a ground-state regardless of signs of spins σ_C since the flip of σ_C does not alter the energy. This leads to the degeneracy $2^{N/3}=\exp[(N/3)\ln 2]$, giving the entropy per spin $s=(\ln 2)/3\simeq 0.231$ (throughout we set the Boltzmann constant unity). The actual entropy density has been analytically calculated by Wannier [2] as

$$s=(2/\pi)\int_0^{\pi/3}\ln(2\cos x)\,dx\simeq 0.3383, \tag{5}$$

which is larger than the above estimate due to the existence of further freedom [2] for the spin flip.

Since the lattice geometry forbids a unit plaquette to have only two unsatisfied bonds, the configuration of the lowest excitation in each unit plaquette is that in which all three bonds are unsatisfied (i.e., all three spins with up or down simultaneously). This unit plaquette serves as a localized lowest excitation, a point defect [18]. It is easy to see that a single defect alone cannot be annihilated by flip of one of the spins forming that defect. Annihilation of defects occurs in pairs. Hence a ''charge'' can be assigned to the defect. The positive charge defect can be identified with *up-plaquette* (i.e., triangle pointing upward) with all three spins having the same sign (regardless of up or down) and the negative charge defect with *down-plaquette* (i.e., triangle pointing downward) with all-the-same-sign spins. As we see shortly below, the coarsening process here involves motion and annihilation of these defects (with opposite charges) which are created in a random disordered initial state. A defect can move one unit lattice spacing by the flip of one of the spins belonging to that defect. The motion of a defect does *not* cost energy. The energy is lowered only via annihilation of defects.

In addition to the presence of the point defects, there exists another object called the loose spin [19] which plays an important role in the coarsening dynamics. The loose spin is defined as the spin whose six nearest neighbors are changing their signs alternatingly (see Fig. 2). Hence the loose spin can be flipped without energy cost. Of course its existence is intimately connected to the infinite degeneracy of the ground-state. Flips of loose spins provide incessant fluctuations in the background ground-state configurations. As discussed in detail below, these ''vacuum'' fluctuations are strongly correlated with defect motions.

Shown in Fig. 3(a) is the relaxation of the defect density $\rho(t)$ and the excess energy $\Delta E(t)\equiv E(t)-E_G$ with $E_G=-1$ being the equilibrium ground-state energy per spin. They are shown to be proportional to each other, as they should be in the zero-temperature quench. Figure 3 shows that they exhibit a power-law relaxation $\Delta E(t)\sim\rho(t)\sim t^{-\psi}$ with $\psi\simeq 0.86(2)$. This relaxation is due to annihilation of defects. Note that the relaxation of the defect density is related to the growing length scale $\xi(t)$ extracted from the dynamic spin-correlation function as $\rho(t)\sim\xi^{-2}(t)$, as demonstrated in the inset of Fig. 3(a). This result implies that the spin-correlation length $\xi(t)$ is proportional to the average separation $d(t)$ between defects since $\rho(t)\sim d^{-2}(t)$ in two spatial dimensions, assuming uniform distribution of defects. Therefore we may state that the coarsening process is governed by the single length scale which is the growing separation between defects due to their mutual annihilation.

We point out that the above subdiffusive domain growth is *not* in accord with a recent simulation work of Moore

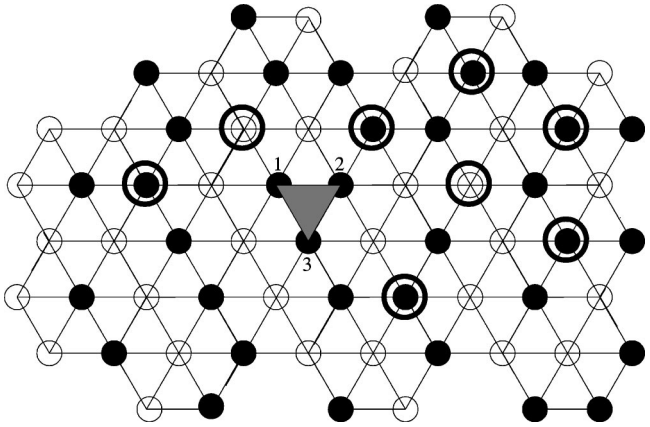

FIG. 2. A negative charge defect and its nearby spin configuration which is extracted from a simulation run. Filled and open circles represent up and down spins, respectively. Encircled are the loose spins which can be flipped without energy cost. Fluctuation of the background state is mediated by flips of these loose spins. When either spin 2 or spin 3 is flipped, the defect moves horizontally to the right or along the direction $(2\rightarrow 3)$ downward, respectively, whereas the defect is not allowed to move (to the left) by the flip of spin 1 which requires creation of defects. However, if the loose spin which is one of the nearest neighbors to spin 1, is flipped, then the flip of spin 1 lets the defect move to the left. Hence for this case the background fluctuation opens up the previously blocked path of the defect. The opposite event occurs for spin 2 where the flip of the loose spin (nearest to spin 2) blocks the previously allowed path of the defect. On average, the loose spin fluctuations enhance the diffusion of defects compared to the case of frozen background.

et al. [14]. They claim that the relaxation of the defect density exhibits a logarithmic correction to the diffusive behavior as $\rho(t)\sim(\ln t)/t$. Their interpretation on their simulation results is that the defect motion in the present system is exactly the same as that observed in the coarsening of the two-dimensional XY model [20–22]. In particular, they assert that the defects interact with Coulomb force and the mobility of defects depends on the distance between them. The only difference is the origin of the Coulomb force: it is argued to be entropic rather than energetic since the motion of defects does not cost energy. With our simulation data, we have tried to check if the defect density has the logarithmic correction as $\rho(t)\sim(\ln t)/t$ by plotting $t\rho(t)$ against $\ln t$. As shown in Fig. 3(b), instead of a straight line, we find that the plot shows a considerable upward curvature in almost the entire simulation time up to $t=81\,920$ (MCS). This is in contrast to the result of Moore *et al.*, which claims that there exists a time region of about one and half decade with a straight line fit. But we do not see such a regime in our simulations. It is not clear to us what causes this discrepancy between the two simulation results. One comment we can make is that since we are dealing with a dominant multiplicative correction factor, the correction fit usually should work for almost entire time region. The result of Moore *et al.*, showing only a one and half decade of logarithmic correction fit after three and half decades of something else, seems very unusual in that sense. We even do not see such a regime in our simulation but rather find a continuous upward curvature. Although in

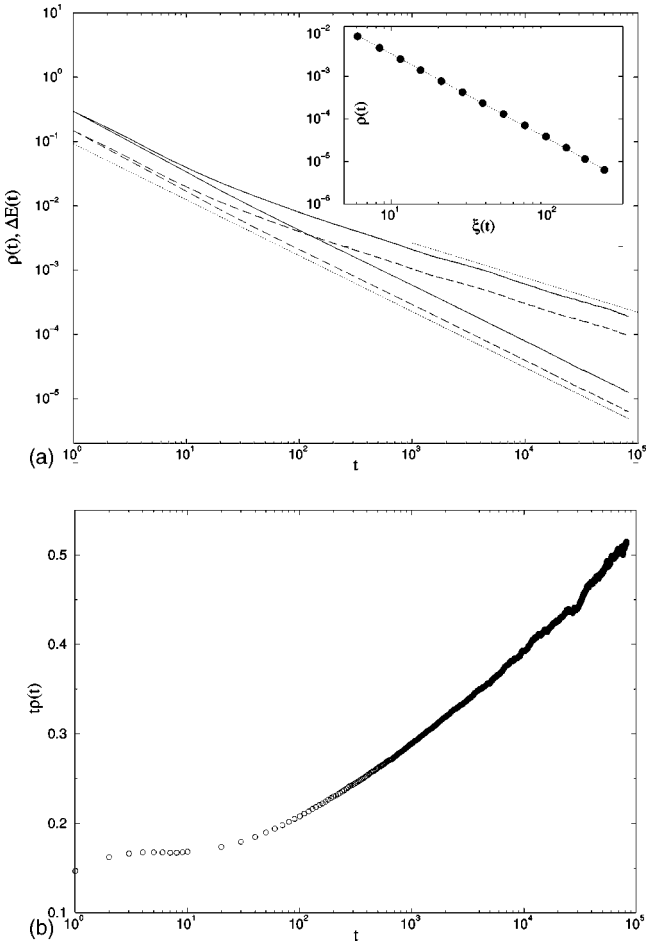

FIG. 3. (a) The relaxation of the defect density $\rho(t)$ (dashed line) and the excess energy density $\Delta E(t)$ (solid line) for the two cases of fluctuating and frozen background ground-state. Slower decay in long times is for the latter case. Dotted lines are the straight lines with slopes -0.86 and -0.56. Inset: A double log plot of $\rho(t)$ vs $\xi(t)$. The filled circles are the simulation data, and dotted is the straight line with slope -2. (b) A semilog plot of $t\rho(t)$ vs t. It shows a continuous upward curvature over almost the entire time region.

principle we cannot rule out the possibility that our data cross over to a straight line for $t>81\,920$ (MCS), it would be an extremely odd scenario that the defect density in this model shows a power law over almost five decades of time scale and then change its behavior for longer times. At any rate, in our simulation time scales, the power-law decay appears to be far superior to the power law with logarithmic correction.

We find that the actual defect motion is highly correlated with its local environment. This is one crucial feature of the defect dynamics in the present system, which we believe to be the microscopic origin of the observed subdiffusive dynamics of defects. Figure 2 shows a single defect and its local environment which is extracted from a real simulation run. We observe that among six possible directions of defect motion there exist some directions along which the motion of defect is not allowed at least momentarily until neighboring spins provide favorable environment by spin flip. This temporary blocking of defect motion is due to the fact that a spin

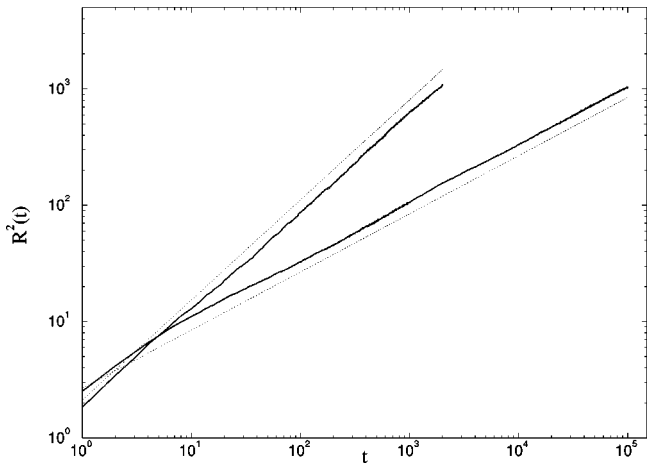

FIG. 4. Mean-square displacement $R^2(t)$ vs time t for an isolated defect for the two cases of fluctuating and frozen background ground-state. The solid line with larger slope is obtained for the former case. Dotted lines are the straight lines with slopes 0.86 and 0.50.

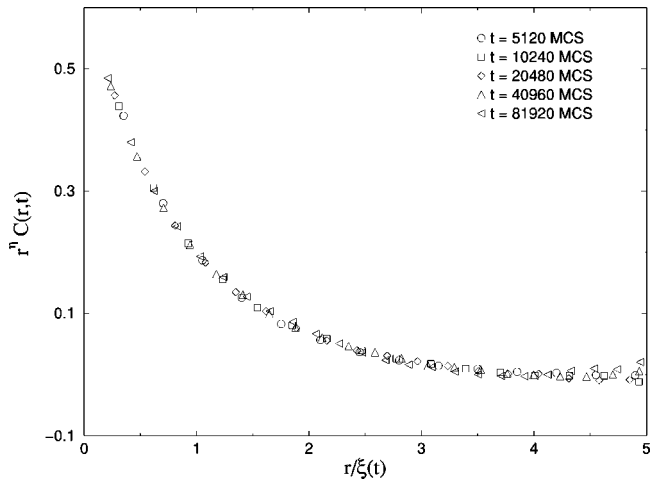

FIG. 5. Scaling collapse of the spin-correlation function $C(r,t)$ when fluctuations of loose spins are completely suppressed. The same critical dynamic scaling as in Fig. 1(a) holds in this slow dynamics as well. But the scaling function is distinct from the one shown in Fig. 1(a).

flip leading to the defect motion along the blocked direction requires creation of defect pairs, which is forbidden in the zero-temperature kinetics. Note that this microscopic blocking effect is absent, for example, in the zero-temperature coarsening of one-dimensional ferromagnetic Ising model where a defect (domain wall) performs free random walk, which exhibits a diffusive domain growth. In connection with this feature, we have calculated the time dependence of the mean-square displacement of defects in the low-density limit, i.e., when defects are far away from one another. We find that $R^2(t) \sim t^{\beta}$ with $\beta \simeq 0.86$, as shown in Fig. 4. Therefore the RMS displacement exactly corresponds to the growth law of interdefect separations and also the scale of domain size $\xi(t)$. This provides a strong numerical evidence that defects which are sufficiently isolated perform restricted (subdiffusive) random walks, which is most probably due to the fluctuation of ground-states. If, on the other hand, we suppose that a kind of entropic Coulomb force is in action here for the motion of defects, the local motion of a sufficiently isolated defect should be at least diffusive, which then is inconsistent with our result above. Therefore we believe that the consistent picture on the defect dynamics is that defects perform a restricted random-walk motion due to their correlated motion with loose spins, not due to their presumed Coulombic interaction.

We further point out that the ground-state fluctuations actually facilitate the motion of defects; that is, the dynamics becomes much slower in the absence than in the presence of background fluctuations of loose spins. We let the background ground-state configuration fixed during the coarsening and let the defect motion occur by flipping the spins on the triangular plaquette pertaining to the defects. In this situation, the RMS displacement of a defect in the low-density limit shows $R^2(t) \sim t^{\beta}$ with $\beta \simeq 0.50$, as shown in Fig. 4. We therefore see that fluctuations of ground-state configurations have the effect of relatively enhancing the diffusion of defect motion compared to that in the absence of the fluctuations. We have also calculated the dynamic spin-correlation function $C(r,t)$ in this situation. As shown in Fig. 5, we find that $C(r,t)$ shows the same critical dynamic scaling as in Eq. (4), but with a much smaller growth exponent; $C(r,t) = r^{-\eta} G(r/t^{1/z})$ with $1/z \simeq 0.18$. As for the defect relaxation, we find that the defect density relaxes as $\rho(t) \simeq t^{-\psi}$ with $\psi \simeq 0.56$. Thus not only the dynamics slows down but also it becomes more complicated: it is characterized by appearance of the multiple length scales. Further study is needed to resolve this puzzling nonproportionality among these length scales in the absence of loose spin fluctuations.

Phenomenologically one may imagine that the fluctuations of ground-state configurations among different degenerate states supply some background potential with spatial dependence. We have seen that these fluctuations cause a microscopic blocking on the motion of defects. In general, one thus might represent this kind of restricted potential as some spatio-temporally correlated noise upon the motion of defects as follows: $d\vec{r}(t)/dt = \vec{f}(r,t)$ where $\vec{r}(t)$ is the position vector of the defect and $\vec{f}(\vec{r},t)$ is a spatiotemporally correlated noise with $\langle \vec{f}(\vec{r},t) \cdot \vec{f}(\vec{r}',t') \rangle = \mathcal{F}(|\vec{r}-\vec{r}'|, |t-t'|)$. If we suppose that $\mathcal{F}$ is replaced by a simple uncorrelated white noise, then we would get a pure diffusion. Since the numerical result from Monte Carlo dynamics shows a subdiffusive behavior, we expect that some spatiotemporal correlation in the noise should be taken into account. This is also reasonable considering the fact that the zero temperature is the critical point with power-law correlations in the spin fluctuations. Since we do not know of any analytic methods of solving the above effective equation for the motion of an isolated defect, we cannot give any concrete functional form of $\mathcal{F}$. The full set of higher moments of the displacement $(\vec{r}-\vec{r}')$ may be needed to determine these exponents.

We have also considered the relaxation *within* the ground-state manifold. Before we discuss the dynamics, we briefly describe the method of dimer coverings [15] which provides an effective way of classifying the macroscopically large degenerate ground-states. In a ground-state configuration, one

draws a line (dimer) such that it intersects every unsatisfied bond from one center to the other of the two unit plaquettes sharing this unsatisfied bond. Then each unit plaquette in a ground-state configuration has only one dimer. By the same procedure, one can obtain another dimer covering for a ''standard'' ground-state. This standard ground-state is chosen to be the ''stripe'' state in which every spin along each horizontal direction has the same sign, but the sign is alternating along the vertical direction. For this stripe state the dimer is then placed vertically on every horizontal bond. Superposing a dimer configuration onto the standard one, one obtains a string configuration. The ground-states can be classified by their string density which is defined as $p \equiv N_s/L$ where N_s is the number of strings and L linear dimension of the system. The ground-states can be arranged into different sectors and each sector is characterized by the string density p of the ground-states belonging to that sector. The entropy density $\mathcal{S}(p)=(1/N)\ln\mathcal{N}(p)$ [$\mathcal{N}(p)$ being number of ground-states] in a given sector with the string density p has been exactly calculated by a transfer-matrix method as [13]

$$\mathcal{S}(p)=(2/\pi)\int_0^{p\pi/2}\ln(2\cos x)dx. \tag{6}$$

We thus know that the sector with nonzero string density has a macroscopic number of ground-states: the degeneracy loses its extensivity only for $p=0$. Stripe state is an example of such zero-entropy states. The entropy function $\mathcal{S}(p)$ has the maximum at $p=2/3$ since $\mathcal{S}'(p)=\ln[2\cos(p\pi/2)]=0$ at $p=2/3$ and $\mathcal{S}''(p)=-(\pi/2)\tan(p\pi/2)<0$ for $0<p\leqslant 1$ where $'\equiv d/dp$. The maximum entropy of $\mathcal{S}(p)$ at $p=2/3$ is nothing but the entropy density s derived by Wannier, Eq. (5). Considering the exponential dependence on N of the degeneracy, this implies that the sector with $p=2/3$ dominates in its number of ground-states. Note also that the loose spins are always located at *isolated* kinks on a string.

Now we discuss the relaxation dynamics within the ground-state manifold. In particular, we have taken initial state as the one shown in Fig. 6. We consider the zero-temperature quench from this state. The string density for this state is $p=2/3$. Thus it belongs to the maximum entropy sector. The density of loose spins is also given by 2/3. Each sublattice in this state has a long-range order with the sublattice magnetization $m_s=1$. Since the initial state is defect free, the time evolution from this state does not proceed via mutual annihilation of defects. As observed from Fig. 6, the relaxation must proceed via flip of loose spins. Therefore the initial defect-free ground-state without loose spins cannot relax at all; the relaxation is trivially nonergodic. The stripe state is such an example. Once a loose spin is flipped, then its six nearest neighbors cannot relax due to nonzero energy cost for the flip. Another loose spin which is not a nearest neighbor to the flipped loose spin should be flipped for further relaxation. It would be then crucial that the once-flipped loose spins be selected *again* since the relaxation would be frozen without these repeated flips.

Shown in Fig. 7(a) is the dynamic spin-correlation function $C(r,t)$ where spins belong to the same sublattice.

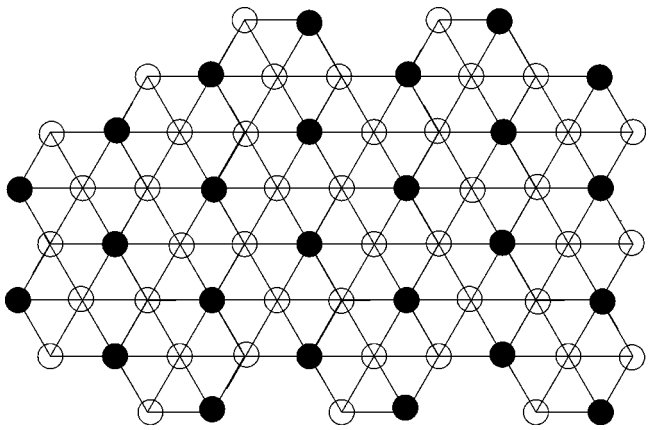

FIG. 6. A ground state with fully ordered sublattice magnetization ($m_s=1$). This state has the loose spin density $\rho_l=2/3$ (open circles are loose spins) as well as the string density $p=2/3$ (see the text for detail). The time evolution from this state is driven by fluctuations of loose spins.

$C(r,t)$ for a given time t relaxes toward a nonzero value at large distances which is the square of sublattice magnetization, $m_s^2(t)$. This saturation sets in at larger distances for late times; that is, the correlation length $\xi(t)$ of spin fluctuation grows in time, which is the only growing length scale in the relaxation. This relaxation also exhibits a critical dynamic scaling when the distance is rescaled in terms of $\xi(t)$, which is demonstrated in Fig. 7(b) for the correlation function for the spin fluctuation $C_F(r,t)\equiv[C(r,t)-m_s^2(t)]$. The scaling function is distinct from that for disordered initial state. Inset of Fig. 7(b) shows that the correlation length of spin fluctuation exhibits a diffusive growth in time, i.e., $\xi(t)\sim t^{1/z}$ with $1/z\simeq 0.51$. One can also directly obtain the dynamic exponent z from the relaxation of the sublattice magnetization $m_s(t)$. From the critical dynamic scaling $C(r,t)=r^{-\eta}Q(r/\xi(t))$, the scaling function $Q(x)$ should behave as $Q(x)\sim x^{\eta}$ for large x. This gives $m_s^2(t)\sim\xi^{-\eta}(t)\sim t^{-\eta/z}$. Inset of Fig. 7(a) shows a power-law relaxation of $m_s^2(t)$. The measured slope -0.25 gives the dynamic exponent $z\simeq 2.0$. This dynamic exponent, indicating a diffusive growth, is *smaller* than the corresponding one for random disordered initial configuration. Therefore we observe that there exists an intriguing initial-state dependence in the nonequilibrium critical dynamics of the present system. A similar initial-state dependence has been observed [16] in the nonequilibrium critical dynamics of the two-dimensional XY model quenched to any temperature on the critical line of temperatures $0<T\leqslant T_{KT}$, T_{KT} being the Kosterlitz-Thouless (KT) transition temperature. In this case, while the nonequilibrium correlation length $\xi(t)$ exhibits a diffusive growth $\xi(t)\sim t^{1/z}$ with $z=2$ for an ordered initial state without free vortices, $\xi(t)$ grows in time as $\xi(t)\sim(t/\ln t)^{1/2}$ for a disordered initial state with free vortices.

Another related quantity we attempted to monitor is the time dependence of the loose spin density $\rho_l(t)$ which is shown in Fig. 8. Its relaxation is characterized by a power-law approach to the nonvanishing asymptotic value $\rho_l(\infty)\simeq 0.29$ for an ordered initial state [$\rho_l(0)=2/3$] of Fig. 6:

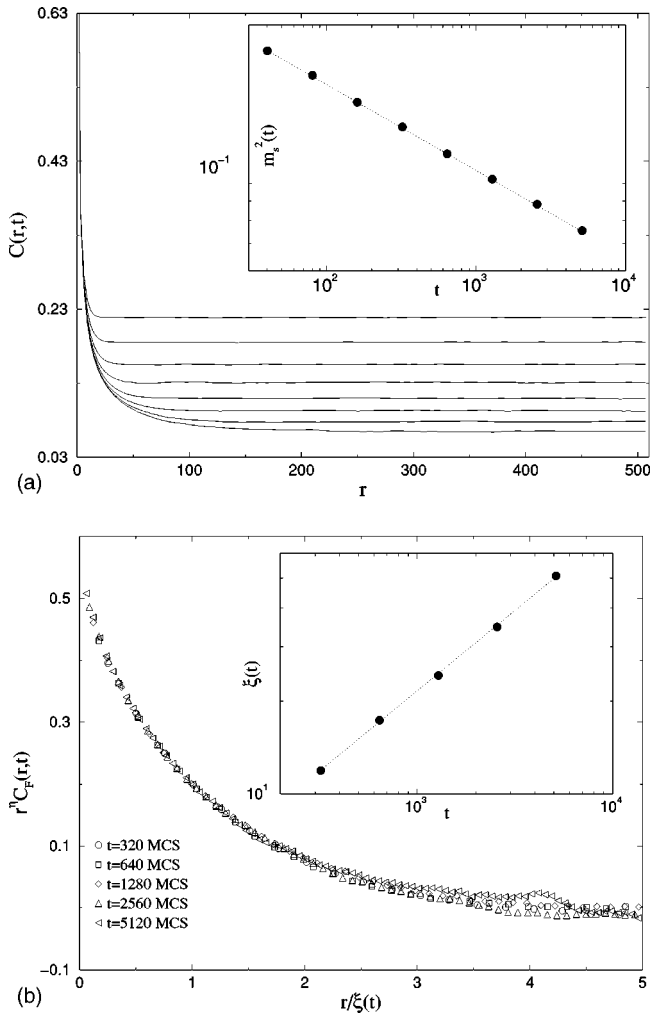

FIG. 7. (a) Equal-time sublattice spin-correlation function $C(r,t)$ for the dynamics within the ground-state manifold from an initial state shown in Fig. 6. The times are given by $t=40\times 2^n$ MCS with $n=1,2,\ldots,8$. The spin correlation at long distances is equal to $m_s^2(t)$ which decays in time. Note that the distance over which $C(r,t)$ relaxes to its asymptotic value becomes larger with increasing time, which means that the correlation length of the spin *fluctuation* [shown in (b)] grows in time. Inset: Relaxation of the sublattice magnetization square $m_s^2(t)$. The dotted line is a straight line with slope -0.25. Using a scaling argument $m_s^2(t)\sim t^{-\eta/z}$ and $\eta=1/2$ we estimate the dynamic exponent $z\simeq 2.0$. (b) A scaling plot for $C_F(r,t)$. The nonequilibrium dynamics within the dominant sector (see the text) from the initial state shown in Fig. 6 also satisfies a critical dynamic scaling property. Inset: Growth of the correlation length $\xi(t)$ for the spin-fluctuation correlation function $C_F(r,t)\equiv C(r,t)-m_s^2(t)$. Dotted line is a straight line with slope 0.51.

$\rho_l(t)=\rho_l(\infty)+At^{-\alpha}$ with $\alpha\simeq 0.98$ (A is a positive constant). We also find that starting from a random disordered state with defects $\rho_l(t)$ shows a different power-law relaxation toward the *same* asymptotic value $\rho_l(\infty)$: $\rho_l(t)=\rho_l(\infty)-Bt^{-\alpha'}$ with $\alpha'\simeq 0.75$ and B being a positive constant. The slower relaxation seems to be due to the presence of defects. So far there is no known relationship between the growth exponent and the above two exponents α and α'. These two

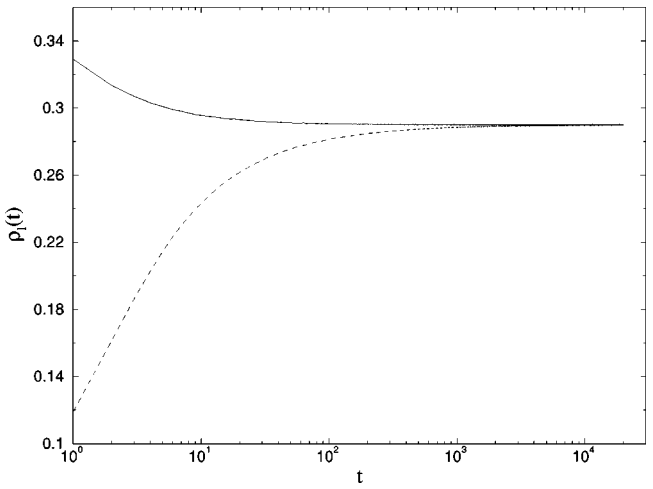

FIG. 8. Relaxation of the loose spin density $\rho_l(t)$ for both cases of the initial state of Fig. 6 (solid line) and random disordered initial states with defects (dashed line). For both cases $\rho_l(t)$ approaches toward the same asymptotic value $\rho_l(\infty)\simeq 0.29$.

exponents are probably new nonequilibrium exponents in the triangular AFI model. The same asymptotic value of $\rho_l(\infty)$ for two different types of initial states indicates that random initial states evolve toward the dominant sector which has maximum entropy, i.e., the sector with the string density $p=2/3$. Thus the sector with string density $p=2/3$ has the average loose spin density $\rho_l\simeq 0.29$. This asymptotic value is perhaps related to the entropy of the ground-state. It would be interesting to verify this possibility analytically. Note also from Fig. 6 that it is not possible to increase or reduce the number of strings by flips of loose spins. Thus the evolving states always lie within the same sector. This feature should hold for an initial state with a general string density p. In this sense, the time evolution within the ground-state manifold is a nonergodic dynamics. Recent works have considered an entropy vanishing transition from the dominant sector to the zero-string sector within the ground-state manifold which is induced by an external perturbation such as lattice compression [12] or staggered field [13]. This entropy vanishing transition is similar to that observed in the random energy model and other so-called discontinuous spin-glass models, which may have significant implications to the nature of the liquid-glass transition [23].

In connection with the nonergodic relaxation within the ground-state manifold, it is known that the deposition and evaporation kinetics of k-mers ($k\geqslant 3$) on one-dimensional lattice is characterized by the break up of the entire phase space into the exponentially large number of sectors in the steady state [24]. Each sector has a unique value of the so-called irreducible string which is a conserved quantity during the time evolution [25,26]. Hence the dynamics is strongly nonergodic since sectors are mutually disconnected. Moreover, it is shown numerically that the spin (particle) autocorrelation in a sector shows either a power-law or a stretched exponential relaxation, the power-law exponent being sector dependent. It would be therefore very interesting to examine the interesting possibility that the dynamic exponent z characterizing the correlation of spin fluctuations may differ from one sector to another in the present system.

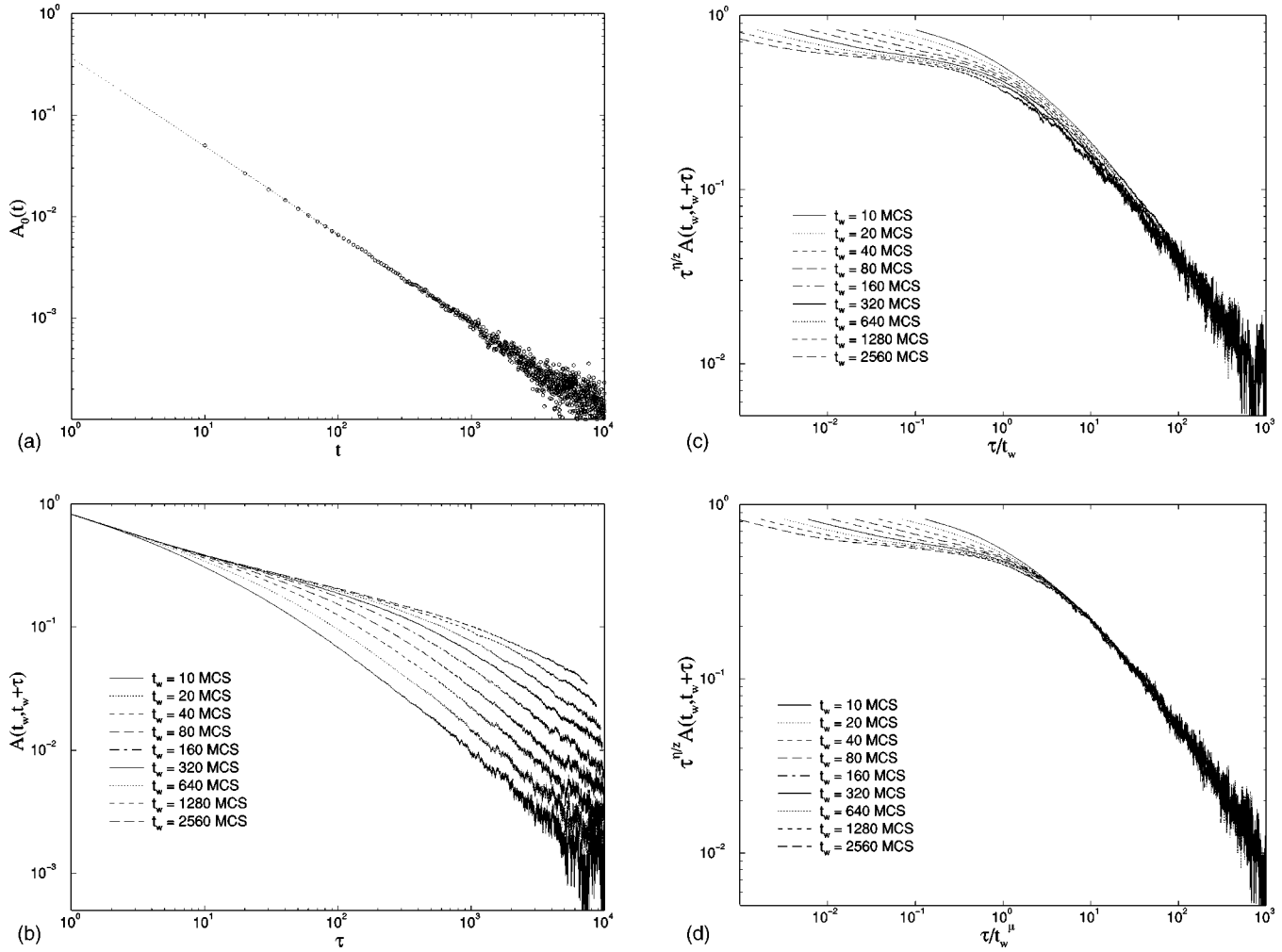

FIG. 9. (a) Relaxation of the spin-autocorrelation function $A_0(t)$. Solid line is a straight line with slope 0.86. (b) Relaxation of the two-time spin-autocorrelation function $A(t_w,t_w+\tau)$ for various waiting times. (c) Scaling attempt according to $A(t_w,t_w+\tau)=\tau^{-\eta/z}\mathcal{G}(\tau/t_w)$. This scaling scheme is based on the assumption that the relaxation time is linearly proportional to the waiting time t_w. This simple scaling is shown to break down in long times. (d) Scaling attempt according to $A(t_w,t_w+\tau)=\tau^{-\eta/z}\mathcal{G}(\tau/t_w^{\mu})$ with $\mu=0.89$. This scaling scheme takes into account the fact that the relaxation time increases with slower rate than the waiting times. This scaling scheme yields a better data collapse than in (c) in long times.

We now turn our attention to the properties of temporal correlations in the coarsening process. In the present work, we limit ourselves to the quench from disordered initial states. We consider the two-time spin-autocorrelation function

$$A(t_w,t_w+\tau)=\frac{1}{N}\left\langle \sum_{i=1}^{N}\sigma_i(t_w)\sigma_i(t_w+\tau)\right\rangle, \qquad (7)$$

which measures temporal correlation of spin configurations at two different times during the evolution process. By setting the waiting time t_w zero, i.e., $t_w=0$, one obtains the usual nonequilibrium spin-autocorrelation function $A_0(t)\equiv A(0,t)$ which measures relaxation of the correlation with the initial state. It is known [27] that $A_0(t)$ in typical ordering kinetics exhibits a power-law relaxation as $A_0(t)\sim\xi^{-\lambda}(t)$ where the exponent λ is a new nonequilibrium exponent. For example, for nonconserved ferromagnetic Ising models with spin-flip kinetics, which are quenched to the critical temperature T_c, analytic result [28] shows that $\lambda=d$ (d being the spatial dimension) for one spatial dimension. For higher dimensions $d=2,3$, it is found numerically [29] that $\lambda<d$ for the quench to T_c. It is also known [30] that $\lambda=d$ for the conserved spin-exchange kinetic Ising model in two dimensions quenched to T_c. For our case Fig. 9(a) shows that it relaxes as $A_0(t)\sim t^{-0.86}$, which implies $\lambda\simeq 2.0$. It is quite probable that λ is exactly equal to the spatial dimension of the system, i.e., $\lambda=d=2$. To the best of our knowledge, the present model is the only system with nonconserved kinetics which gives $\lambda=d$ in the spatial dimension higher than one.

Shown in Fig. 9(b) is $A(t_w,t_w+\tau)$ versus τ for various waiting times. For short time regions $\tau/t_w\ll 1$, the relaxation does not depend on the waiting times. This is in accord with the general expectation that the relaxation in the short-time region is like the equilibrium one where the time-translation

invariance holds. On the other hand, $A(t_w, t_w+\tau)$ exhibits a strong waiting time dependence for large τ region: the relaxation becomes slower for longer waiting times. This nonequilibrium behavior is known as the aging phenomenon. It would therefore be very interesting to investigate the possible scaling behavior of this aging region. Usually, the two-time spin-autocorrelation function in the critical quench satisfies the scaling form [31,32]

$$A(t_w, t_w+\tau) = \tau^{-\eta/z}\mathcal{G}(\tau/t_w), \tag{8}$$

where the factor $\tau^{-\eta/z}$ is needed since $A(t_w, t_w+\tau)$ should recover the equilibrium critical relaxation $\tau^{-\eta/z}$ in the limit $t_w \to \infty$ [therefore $\mathcal{G}(0)$ should be a constant]. The above scaling implies the relaxation time grows *linearly* with the waiting time. However, to our surprise, this scaling form does not hold for the present system, as shown in Fig. 9(c). Instead, we find that the following generalized scaling form

$$A(t_w, t_w+\tau) = \tau^{-\eta/z}\mathcal{G}(\tau/t_w^{\mu}), \quad \mu \simeq 0.89, \tag{9}$$

is obeyed in the aging regime, as demonstrated in Fig. 9(d). The value of μ smaller than unity implies that the relaxation time increases with a slower rate than the waiting time. This type of scaling may be termed as a critical subaging behavior. This is a unique feature of the present model which has not been observed in other usual nonequilibrium critical relaxations.

Another interesting aspect in the nonequilibrium coarsening dynamics is the ''persistence'' of the system. The persistence can be quantified as the time dependence of the probability that a spin maintains its initial state (i.e., a spin is never flipped in the case of spin-flip kinetics) up to time t. It was found for a variety of spin systems quenched to $T=0$ that the fraction of the persistent spins (or the persistent probability) exhibits a power-law relaxation $\rho_p(t) \sim t^{-\theta}$ where the exponent θ is another new exponent which is not related to other exponents characterizing the coarsening dynamics. The exponent θ has been analytically or numerically computed. For example, in the zero-temperature dynamics of the ferromagnetic Ising model, $\theta = 3/8 = 0.375$ (analytic) for $d=1$ [33], and $\theta \simeq 0.22$ (numerical) for $d=2$ [34].

In order to compute the density of persistent spins, we first define for each site i the quantity $n_i(t) \equiv \frac{1}{2}[1 + \sigma_i(0)\sigma_i(t)]$ [35]. Initially ($t=0$), $n_i(0)=1$ for all sites. If the spin at site i is once flipped at time t, then $n_i(t)=0$. Once it becomes zero, then the value is made unchanged regardless of subsequent flips of the same spin $\sigma_i(t)$. Therefore the density of persistent spins $\rho_p(t)$ is just the number of sites with $n_i(t)=1$ at time t divided by N, i.e., $\rho_p(t) = \langle \Sigma_{i=1}^{N} n_i(t) \rangle / N$. Figure 10(a) shows a semilog plot of $\rho_p(t)$ versus t. We first see that there is a very fast relaxation in the short-time region $t \lesssim 50$. This early time relaxation may be closely related to the rapid initial annihilation process of defects immediately after the quench. After this transient relaxation, $\rho_p(t)$ shows an exponential decay in time region $t \gtrsim 50$, as demonstrated in Fig. 10(a). This exponential relaxation is due to the dominant contribution of the loose spins which make ceaseless flips.

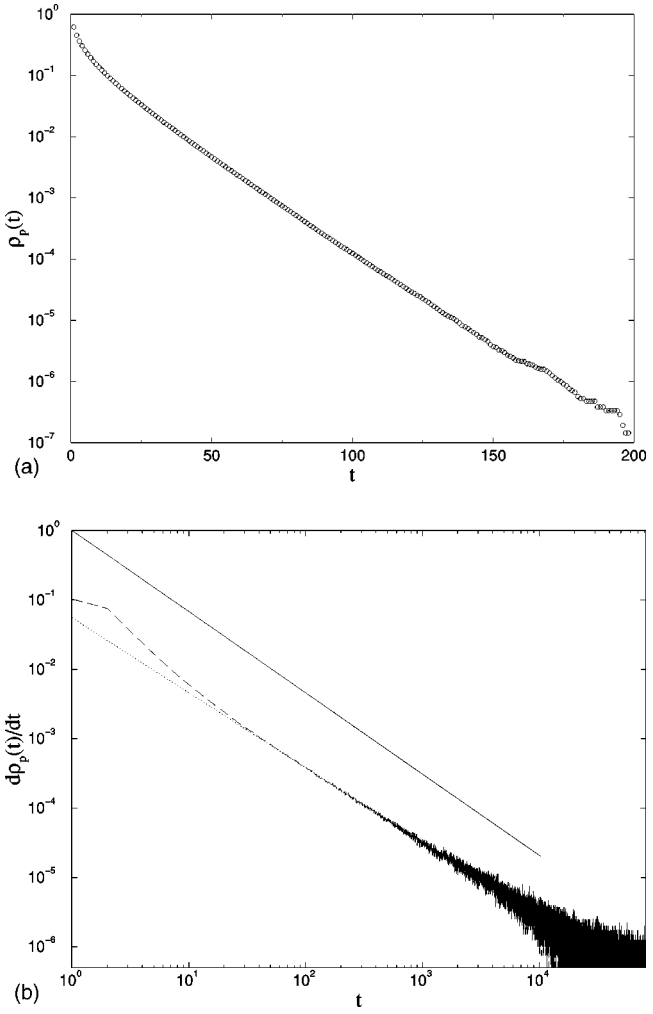

FIG. 10. (a) The persistence probability $\rho_p(t)$ vs t in a semilog plot. After a very fast transient relaxation ($t \lesssim 50$), $\rho_p(t)$ exhibits an exponential decay. (b) Time derivative of the persistence probability for the two different cases. The solid line is the time derivative of the persistence probability obtained when fluctuations of loose spins are ignored in counting persistent spins, and only spin flips involving defects are taken into account. Dashed line is the time derivative of the persistence probability obtained when fluctuations of loose spins are forced to be frozen in the time evolution. These two cases lead to different power-law relaxations. Dotted line is a straight line with slope -1.08. The solid line has the slope -1.17. This means that $\rho_p(t)$ approaches its nonvanishing asymptotic value with a power-law relaxation for each case.

One can thus imagine that if we ignore the flipping of the loose spins [i.e., if we set permanently $n_i(t)=1$ for the flip of the loose spins] and if we take into account *only* the flips involving the defect motion and annihilation, one may obtain a qualitatively different result. We have done this and, as shown in Fig. 10(b), indeed instead of an exponential decay we observe a power-law relaxation $\rho_p(t) = \rho_0 + \text{const} \times t^{-\theta}$ with $\theta \simeq 0.17$ where the nonvanishing residual density ρ_0 is due to the fact that flips of loose spins have not been counted. In order to obtain the value of θ, we first differentiate $\rho_p(t)$ to remove ρ_0 and then plot $d\rho_p(t)/dt$ versus t in a double logarithmic scale. The slope, which should be equal

to $-(1+\theta)$, gives the above value of θ. Also shown in Fig. 10(b) is the persistence in the frozen background of ground-state. It appears that $\rho_p(t)$ relaxes toward a nonvanishing value. By differentiating it with respect to t we could estimate the exponent $\theta \simeq 0.08$.

IV. SUMMARY AND CONCLUDING REMARKS

We have considered the nonequilibrium critical dynamics of the triangular AFI model which possesses an exponentially large number of ground-states. Quenched to the zero (critical) temperature from a disordered initial state with many defects, the system relaxes toward the ground-state via mutual annihilation of point defects. Unique feature of the dynamics is, due to a macroscopic degeneracy of the ground-state, that the background ground-state configuration on which defects move *fluctuates* via rapid flips of loose spins. This background fluctuations drive the system into one of the ground state configurations belonging to the maximum entropy sector. At the same time, the fluctuation significantly affects the diffusion property of defect motions. It turns out that the evolution dynamics is characterized by a single dominant length scale, i.e., the mean separation of uniformly distributed defects, and that it is self-similar under the rescaling the distance in terms of this characteristic length scale. We have argued that this length exhibits a genuine subdiffusive growth in time, not a diffusive growth with a logarithmic correction. This subdiffusive nature is attributed to the fact that the structure of the background-state induces a microscopic blocking to the defect motion. The dynamics of defects on a *fixed* background (i.e., loose spins are frozen-in) becomes more subdiffusive compared to that in the presence of background fluctuations.

The nonequilibrium critical dynamics in the present system shows a unique initial-state dependence. This feature is seen when the dynamics within the ground-state manifold is considered. The time evolution within the ground-state manifold proceeds via fluctuations of loose spins. In the present work, we have only considered the dynamics within the dominant ground-state sector, i.e., the sector with the string density $p=2/3$. In particular, the initial state we used was the state with fully ordered sublattice magnetization. The relaxation from this initial state shows a critical dynamic scaling in the spatial spin-correlation function. The correlation length of spin fluctuations exhibits a diffusive growth in time. This kind of initial-state dependence is very unique in that it is absent in the nonequilibrium critical dynamics of typical kinetic Ising models. The only known example where similar initial state dependence is observed is the two-dimensional XY model quenched to $T \leqslant T_{KT}$.

We also observe that the aging phenomenon exhibited by the two-time spin-autocorrelation function defies a conventional scaling scheme based on a simple aging. The two-time spin correlation satisfies a scaling based on the so-called subaging phenomenon (i.e., the slower growth of the relaxation time than that of the waiting time).

Though not presented here, we find that another frustrated Ising system, the fully frustrated Ising model on a square lattice [36] exhibits quantitatively the same critical dynamics as the triangular AFI model [37]. It is well known [19] that these two frustrated systems share common equilibrium properties. We have seen that the two systems belong to the same universality class with respect to the nonequilibrium critical dynamics as well. It would be interesting to study the nonequilibrium critical dynamics of other models which has similar equilibrium properties. The three-state AF Potts model on a square lattice [14,38] is an example of such systems. It would be also valuable to investigate how the nonequilibrium critical dynamics is affected when different dynamic rules such as spin-exchange kinetics are employed.

ACKNOWLEDGMENTS

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